Set-membership identification of Hammerstein systems

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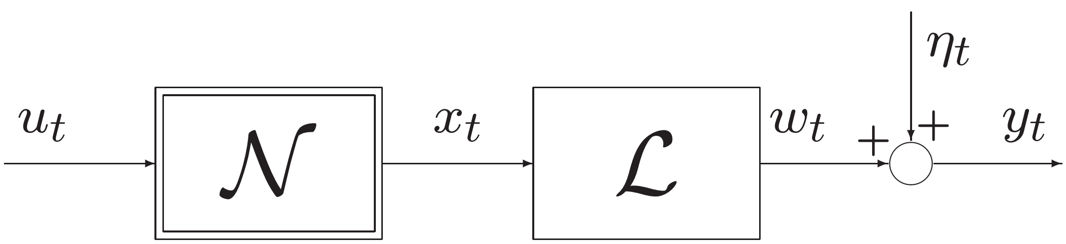
Hammerstein systems

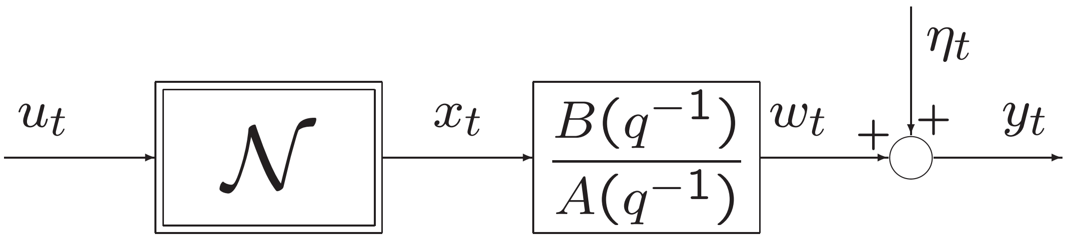
Hammerstein systems are a class of nonlinear block-oriented systems modeled by the following interconnection:



where:

$\mathcal{N}$: memoryless nonlinear gain
$\mathcal{L}$: linear subsystem
x_t: inner signal not measurable

Problem formulation



$$x_t = \sum_{k=1}^{n} \gamma_k \psi_k(u_t)$$

$$A(q^{-1})w_t = B(q^{-1})x_t$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}$$

$$q^{-1} w_t = w_{t-1}$$

Problem formulation

- **Aim:** compute bounds on the parameters $\gamma^{\mathsf{T}} = [\gamma_1\ \gamma_2\ \ldots\ \gamma_n]$ and $\theta^{\mathsf{T}} = [a_1\ \ldots\ a_{na}\ b_0\ b_1\ \ldots\ b_{nb}]$.

- **Prior assumption on the system:**

  1. stability;
  2. n, na and nb are known;
  3. a rough upper bound on the settling time of the system is known.

- **Prior assumption on the measurement uncertainty:**

  1. $\{\eta_t\}$ is UBB: $\|\{\eta_t\}\|_\infty \leq \Delta\eta_t$;
  2. $\Delta\eta_t$ is known;

Proposed solution: preliminary

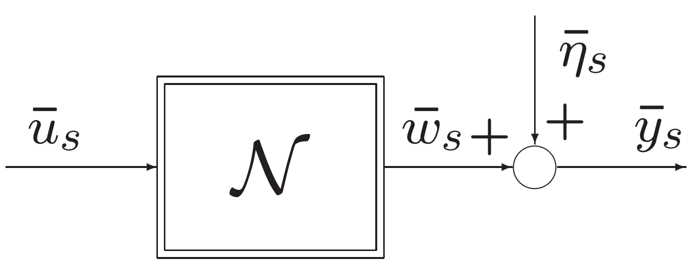
<u>**Remark 1**</u>: The parameterization is not unique

$\Rightarrow$ assume $g = \dfrac{\sum_{j=0}^{nb} b_j}{1+\sum_{i=1}^{na} a_i} = 1$

<u>**Remark 2**</u>: Stimulate the system with step inputs of different amplitudes

$$\Downarrow$$

Steady-state operating conditions:



Proposed solution: first-stage

1) Bounds on the nonlinear block parameters

- Get $M \geq n$ *steady-state* measurements:

  $\bar{y}_s = \sum_{k=1}^{n} \gamma_k \psi_k(\bar{u}_s) + \bar{\eta}_s, \quad s = 1, \ldots, M$

- The (exact) *feasible parameter set* (FPS) $\mathcal{D}_\gamma$, is described by:

$$\mathcal{D}_\gamma = \{\gamma \in R^n : \bar{y}_s = \sum_{k=1}^{n} \gamma_k \psi_k(\bar{u}_s) + \bar{\eta}_s, |\bar{\eta}_s| \leq \Delta\bar{\eta}_s; \; s = 1, \ldots, M\}$$

Remark: $\mathcal{D}_\gamma$ is a convex polytope which provides tight bounds on parameters γ;

Proposed solution: first-stage

Remark: The shape of $\mathcal{D}_\gamma$ may result quite complex for large M and n

$$\Downarrow$$

Compute a tight orthotope outer-bound:

$$\mathcal{B}_\gamma = \{\gamma \in R^n : \gamma_j = \gamma_j^c + \delta\gamma_j, \mid \delta\gamma_j \mid \leq \Delta\gamma_j/2, \quad j = 1, \ldots, n\},$$

$$\gamma_j{}^c = \frac{\gamma_j^{min} + \gamma_j^{max}}{2}$$

$$\Delta\gamma_j = \mid \gamma_j^{max} - \gamma_j^{min} \mid$$

$$\gamma_j^{min} = \min_{\gamma \in \mathcal{D}_\gamma} \gamma_j, \quad \gamma_j^{max} = \max_{\gamma \in \mathcal{D}_\gamma} \gamma_j.$$

Computational aspects: $\mathcal{B}_\gamma$ is obtained solving $2n$ LP problems with n variables and $2M$ constraints.

Proposed solution: second-stage

2) Bounds on the inner signal x_t

Given an input transient sequence $\{u_t\} \in R^N$

$$\Downarrow$$

$$x_t^{min} = \min_{\gamma \in \mathcal{D}_\gamma} \varphi_t^\top \gamma, \quad x_t^{max} = \max_{\gamma \in \mathcal{D}_\gamma} \varphi_t^\top \gamma$$

where:

$\varphi_t = [\psi_1(u_t) \quad \psi_2(u_t) \quad \psi_3(u_t) \ldots \psi_n(u_t)]^\top$

Computational aspects: x_t^{min}, x_t^{max} are obtained solving $2N$ LP problems with n variables and $2M$ constraints.

Proposed solution: second-stage

3) Bounds on the linear block parameters

The input transient sequence $\{u_t\}$ is applied and the (noisy) output sequence $\{y_t\}$ is measured.

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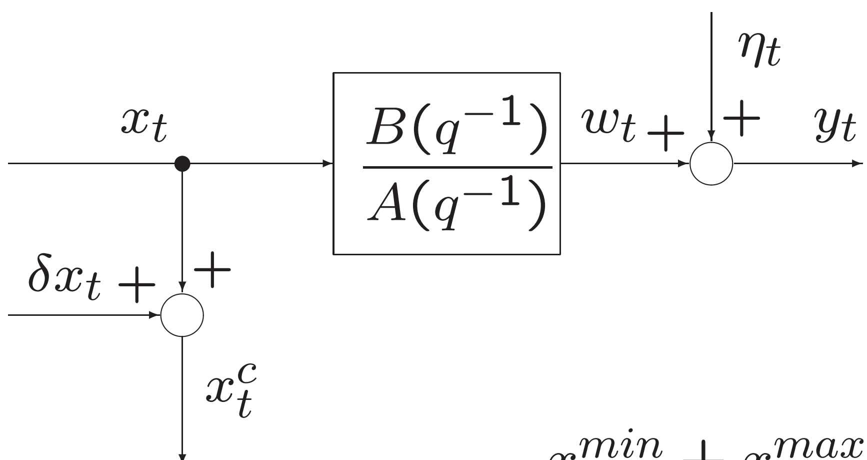
Errors-in-variables (EIV) problem with UBB errors



$$x_t^c = \frac{x_t^{min} + x_t^{max}}{2}$$

$$| \delta x_t | \leq \Delta x_t$$

$$\Delta x_t = \frac{x_t^{max} - x_t^{min}}{2}$$

Proposed solution: EIV problem

Exploiting previous results on static EIV problem with bounded errors
(**Cerone,"Feasible parameter set of linear models with bounded errors in all variables"**, *Automatica '93*)

$$\Downarrow$$

a polytopic outer approximation $\mathcal{D}'_\theta$ of the *feasible parameter set* $\mathcal{D}_\theta$ is characterized by:

$$(\phi_t - \Delta\phi_t)^\mathsf{T}\theta \leq y_t + \Delta\eta_t$$

$$(\phi_t + \Delta\phi_t)^\mathsf{T}\theta \geq y_t - \Delta\eta_t$$

$$\phi_t^\mathsf{T} = \left[-y_{t-1} \ldots - y_{t-na}\; x_t^c\; x_{t-1}^c \ldots x_{t-nb}^c\right]$$

$$\Delta\phi_t^\mathsf{T} = [\Delta\eta_{t-1}\mathrm{sgn}(a_1) \;\; \ldots \;\; \Delta\eta_{t-na}\mathrm{sgn}(a_{na})$$

$$\Delta x_t\mathrm{sgn}(b_0) \;\; \Delta x_{t-1}\mathrm{sgn}(b_1) \;\; \ldots \;\; \Delta x_{t-nb}\mathrm{sgn}(b_{nb})]$$

$$[1 \quad \ldots \quad 1 \quad -1 \quad -1 \quad \ldots \quad -1]\,\theta = -1$$

Proposed solution: EIV problem

Parameter uncertainty intervals $\Delta\theta_j$ are provided by the bounding orthotope $\mathcal{B}_\theta$:

$$\mathcal{B}_\theta = \{\theta \in R^p : \theta_j = \theta_j^c + \delta\theta_j, \mid \delta\theta_j \mid \leq \Delta\theta_j/2, j = 1, \ldots, p\},$$

$$\theta_j^c = \frac{\theta_j^{min} + \theta_j^{max}}{2},$$

$$\Delta\theta_j = \mid \theta_j^{max} - \theta_j^{min} \mid,$$

$$\theta_j^{min} = \min_{\theta \in \mathcal{D}'_\theta} \theta_j, \quad \theta_j^{max} = \max_{\theta \in \mathcal{D}'_\theta} \theta_j.$$

EIV: computational complexity

- **General case:** $\mathcal{B}_\theta$ is obtained solving $2p2^p$ LP problems with p variables and $2N+p+1$ constraints.

- **Remark 1:** If signs of θ_j, $j = 1, \ldots, p$, a-priori known $\Rightarrow$ $2p$ LP problems.

- **Remark 2:** If signs of θ_j not available, $\Rightarrow$ a point estimate (e.g. least squares) will indicate the orthant where the optimization should be carried out. If (some) θ_j^{min} (θ_j^{max})$= 0$, $\Rightarrow$ solve the problem also in the orthants where $\theta_j < 0$ ($\theta_j > 0$).

Example:

$\gamma_1 = 1,\ \gamma_2 = 1,\ \gamma_3 = 1;$
$\psi_1(u_t) = u_t;\ \ \psi_2(u_t) = u_t^2;\ \psi_3(u_t) = u_t^3;$
$A(q^{-1}) = (1 - 1.1q^{-1} + 0.28q^{-2});$
$B(q^{-1}) = (0.1q^{-1} + 0.08q^{-2})$

Signal to noise ratio:

$$SNR = 10 \log \frac{\sum_{t=1}^{N} w_t^2}{\sum_{t=1}^{N} \eta_t^2}$$

$$\overline{SNR} = 10 \log \frac{\sum_{s=1}^{M} \overline{w}_s^2}{\sum_{s=1}^{M} \overline{\eta}_s^2}$$

Bounded relative errors

$\Delta\epsilon^y$ (%)	$\overline{SNR}$ (dB)	True Value	γ_j^{min}	γ_j^c	γ_j^{max}
0.1	61.4	1.000	1.000	1.000	1.001
		1.000	0.999	1.000	1.001
		1.000	0.998	0.999	1.000
1	43.8	1.000	0.990	0.997	1.005
		1.000	0.994	1.001	1.008
		1.000	0.994	1.000	1.007
5	38.1	1.000	0.984	1.019	1.053
		1.000	0.923	0.983	1.044
		1.000	0.938	0.987	1.036
10	25.7	1.000	0.952	1.059	1.166
		1.000	0.816	0.950	1.084
		1.000	0.799	0.919	1.040
20	17.6	1.000	0.893	0.997	1.102
		1.000	0.883	0.994	1.105
		1.000	0.827	0.961	1.095

$\Delta\epsilon^y$ (%)	SNR (dB)	N	True Value	θ_j^{min}	θ_j^c	θ_j^{max}
0.1	64.8	100	-1.100	-1.101	-1.100	-1.098
			0.280	0.278	0.280	0.281
			0.100	0.100	0.100	0.100
			0.080	0.080	0.080	0.080
	64.9	1000	-1.100	-1.100	-1.100	-1.100
			0.280	0.280	0.280	0.280
			0.100	0.100	0.100	0.100
			0.080	0.080	0.080	0.080
1	44.6	100	-1.100	-1.120	-1.103	-1.086
			0.280	0.268	0.282	0.297
			0.100	0.099	0.100	0.101
			0.080	0.076	0.079	0.082
	44.9	1000	-1.100	-1.103	-1.099	-1.096
			0.280	0.276	0.279	0.283
			0.100	0.100	0.100	0.101
			0.080	0.079	0.080	0.081
5	31.3	100	-1.100	-1.160	-1.095	-1.029
			0.280	0.219	0.275	0.331
			0.100	0.091	0.099	0.108
			0.080	0.067	0.080	0.093
	31.0	1000	-1.100	-1.122	-1.103	-1.084
			0.280	0.264	0.282	0.300
			0.100	0.096	0.101	0.106
			0.080	0.076	0.081	0.086

$\Delta\epsilon^y$ (%)	SNR (dB)	N	True Value	θ_j^{min}	θ_j^c	θ_j^{max}
10	24.7	100	-1.100	-1.255	-1.135	-1.014
			0.280	0.208	0.315	0.422
			0.100	0.090	0.106	0.122
			0.080	0.062	0.083	0.104
	24.8	1000	-1.100	-1.167	-1.123	-1.078
			0.280	0.255	0.302	0.350
			0.100	0.097	0.107	0.117
			0.080	0.069	0.081	0.092
20	18.8	100	-1.100	-1.549	-1.181	-0.812
			0.280	0.052	0.370	0.688
			0.100	0.075	0.101	0.128
			0.080	0.027	0.079	0.131
	18.8	1000	-1.100	-1.178	-1.103	-1.027
			0.280	0.209	0.284	0.360
			0.100	0.093	0.105	0.116
			0.080	0.066	0.081	0.095

Bounded absolute errors

$\overline{SNR}$ (dB)	True Value	γ_j^{min}	γ_j^{c}	γ_j^{max}
58.9	1.000	0.996	1.001	1.006
	1.000	0.996	0.998	1.000
	1.000	0.997	1.000	1.002
49.3	1.000	0.989	1.016	1.042
	1.000	0.990	0.996	1.002
	1.000	0.986	0.996	1.006
31.7	1.000	0.914	1.085	1.257
	1.000	0.933	1.003	1.074
	1.000	0.906	0.979	1.052
19.7	1.000	0.822	1.344	1.865
	1.000	0.729	0.899	1.069
	1.000	0.734	0.945	1.156

SNR (dB)	N	True Value	θ_j^{min}	θ_j^{c}	θ_j^{max}
60.0	100	-1.100	-1.104	-1.100	-1.096
		0.280	0.277	0.280	0.284
		0.100	0.100	0.100	0.100
		0.080	0.079	0.080	0.080
59.9	1000	-1.100	-1.102	-1.100	-1.098
		0.280	0.278	0.280	0.282
		0.100	0.100	0.100	0.100
		0.080	0.080	0.080	0.080
47.8	100	-1.100	-1.114	-1.099	-1.084
		0.280	0.264	0.278	0.293
		0.100	0.099	0.101	0.102
		0.080	0.078	0.080	0.083
50.0	1000	-1.100	-1.108	-1.101	-1.094
		0.280	0.274	0.281	0.287
		0.100	0.099	0.100	0.101
		0.080	0.079	0.080	0.081

SNR (dB)	N	True Value	θ_j^{min}	θ_j^{c}	θ_j^{max}
24.8	100	-1.100	-1.210	-1.096	-0.983
		0.280	0.175	0.281	0.387
		0.100	0.094	0.103	0.112
		0.080	0.066	0.084	0.102
29.7	1000	-1.100	-1.168	-1.096	-1.024
		0.280	0.210	0.274	0.338
		0.100	0.094	0.099	0.104
		0.080	0.068	0.081	0.094
13.8	100	-1.100	-1.821	-1.366	-0.911
		0.280	0.077	0.526	0.975
		0.100	0.037	0.092	0.147
		0.080	0.001	0.067	0.133
20.2	1000	-1.100	-1.514	-1.195	-0.877
		0.280	0.074	0.358	0.643
		0.100	0.084	0.105	0.126
		0.080	0.020	0.069	0.119

Conclusions

- The proposed two-stage parameter bounding procedure provides:
  - tight bounds on the parameters of the non-linear block using steady-state input-output data;
  - overbounds on the parameters of the linear part, through the computation of tight bounds on the unmeasurable inner signal x_t;
- The numerical example has showed the effectiveness of the proposed procedure.
- The approach is computationally tractable: the computation related to the above example (n=3, M=10, p=4, N=1000) required few seconds on a standard notebook (AMD 3200)

References

Reference papers for this lesson

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